Does HBT Measure the Freeze-out Source Distribution?

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  - Need a quantum-mechanical description of multiple scattering
- Application of Glauber multiple scattering theory to HBT
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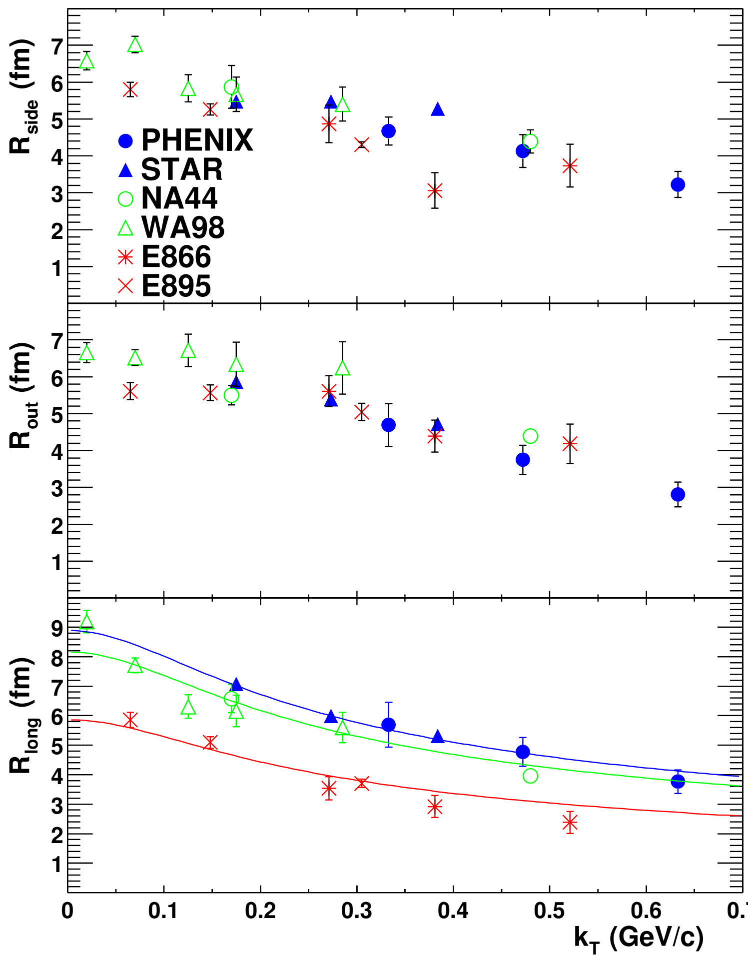
Introduction

There are perplexing puzzles in HBT measurements.

- Relatively small changes of the extracted longitudinal and transverse radii as a function of collision energies
- $R_{\rm out}/R_{side} \approx 1$



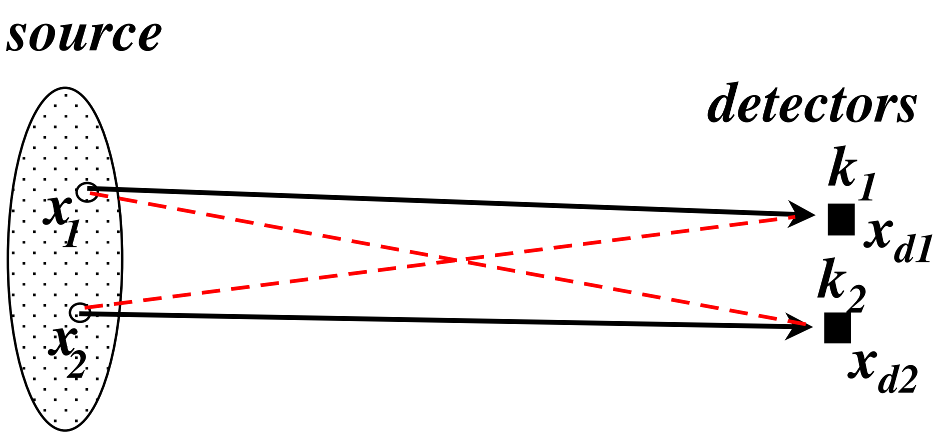
Pb-Pb or Au-Au

Central Collisions

PHENIX	(130 GeV)
STAR	(130 GeV)
NA44	(17.3 GeV)
WA98	(17.3 GeV)
E895	(4.1 GeV)
E866	(4.9 GeV)

K. Adcox *et al.*, Phys. Rev. **88**, 192302 (2002)

Re-examination of HBT conditions



$$P(k_1 k_2) = P(k_1)P(k_2)\,(1 + R(k_1 k_2))$$

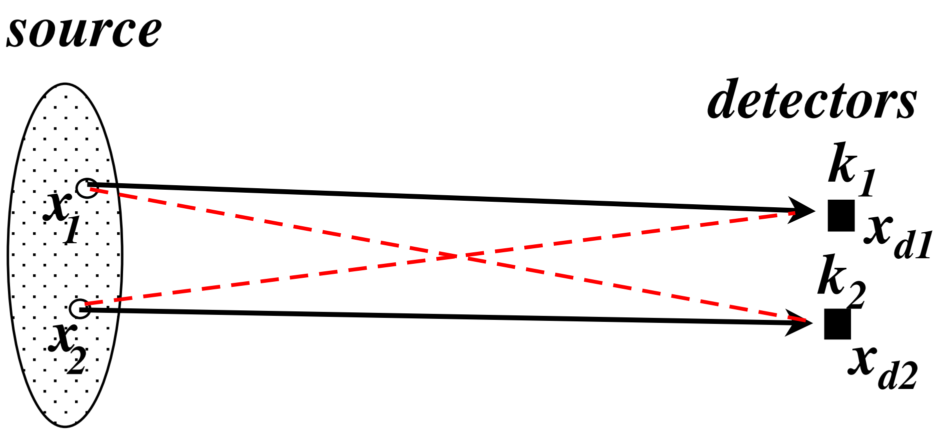
HBT correlation depends on source property

- No HBT correlations ($R(k_1 k_2) = 0$), if the source is coherent
- Presence of HBT correlations ($R(k_1 k_2) \neq 0$), if the source is chaotic

$$P(k_1, k_2) = \frac{1}{2!}\left|\sum_{x_1,x_2} A(k_1 x_1)e^{i\phi_0(k_1 x_1)} A(k_2 x_2)e^{i\phi_0(k_2 x_2)}\psi_{12}(x_1, x_2)\right|^2$$

- No HBT correlations ($R(k_1 k_2) = 0$), if the source is coherent (phase $\phi_0(kx)$ is a simple function of x)
- Presence of HBT correlations ($R(k_1 k_2) \neq 0$), if the source is chaotic ($\phi_0(kx)$ is random and fluctuating)

Coherent source



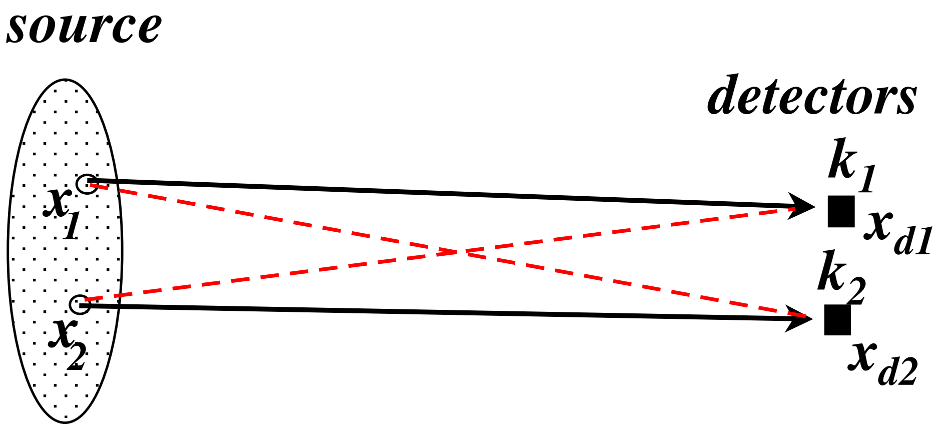
$$P(k_1,k_2) = \frac{1}{2!}\left|\sum_{x_1,x_2} A(k_1x_1)e^{i\phi_0(k_1x_1)}A(k_2x_2)e^{i\phi_0(k_2x_2)}\psi_{12}(x_1,x_2)\right|^2$$

$$\psi_{12} = \frac{1}{\sqrt{2}}\left\{e^{ik_1\cdot(x_{d1}-x_1)+ik_2\cdot(x_{d2}-x_2)} + e^{ik_1\cdot(x_{d1}-x_2)+ik_2\cdot(x_{d2}-x_1)}\right\}$$

No HBT correlation ($R(k_1k_2)=0$) if the phase $\phi_0(kx)$ is a simple functions of x:

$$\begin{aligned}
P(k_1,k_2) &= \frac{1}{2!}\Big|\sum_{x_1,x_2} A(k_1x_1)e^{i\phi_0(k_1x_1)}A(k_2x_2)e^{i\phi_0(k_2x_2)} \\
&\quad\times\frac{1}{\sqrt{2}}\left\{e^{ik_1\cdot(x_{d1}-x_1)+ik_2\cdot(x_{d2}-x_2)} + e^{ik_1\cdot(x_{d1}-x_2)+ik_2\cdot(x_{d2}-x_1)}\right\}\Big|^2 \\
&= \frac{1}{4}\Big|\left(\sum_{x_1} A(k_1x_1)e^{i\phi_0(k_1x_1)}e^{-ik_1\cdot x_1}\right)\left(\sum_{x_2} A(k_2x_2)e^{i\phi_0(k_2x_2)}e^{-ik_2\cdot x_2}\right) \\
&\quad+\left(\sum_{x_2} A(k_1x_2)e^{i\phi_0(k_1x_2)}e^{-ik_2\cdot x_2}\right)\left(\sum_{x_1} A(k_2x_1)e^{i\phi_0(k_2x_1)}e^{-ik_1\cdot x_1}\right)\Big|^2 \\
&= \left|\left(\sum_{x_1} A(k_1x_1)e^{i\phi_0(k_1x_1)}e^{-ik_1\cdot x_1}\right)\left(\sum_{x_2} A(k_2x_2)e^{i\phi_0(k_2x_2)}e^{-ik_2\cdot x_2}\right)\right|^2 \\
&= P(k_1)P(k_2)
\end{aligned}$$

Chaotic source



For a chaotic source, $\phi_0(kx)$ is random and fluctuating,

$$P(k_1,k_2) = \frac{1}{2!}\left|\sum_{x_1,x_2} A(k_1x_1)e^{i\phi_0(k_1x_1)}A(k_2x_2)e^{i\phi_0(k_2x_2)}\psi_{12}(x_1,x_2)\right|^2$$

$$= \sum_{x_1,x_2} A^2(k_1x_1)A^2(k_2x_2)|\psi_{12}(x_1,x_2)|^2$$

$$\psi_{12} = \frac{1}{\sqrt{2}}\left\{e^{ik_1\cdot(x_{d1}-x_1)+ik_2\cdot(x_{d2}-x_2)} + e^{ik_1\cdot(x_{d1}-x_2)+ik_2\cdot(x_{d2}-x_1)}\right\}$$

$$P(k_1k_2) = P(k_1)P(k_2)\left(1+R(k_1k_2)\right)$$

$R(k_1k_2)$ depends on the cross term of $|\psi_{12}|^2$ which contains the phase difference between the two histories

$$k_1\cdot(x_{d1}-x_1)+k_2\cdot(x_{d2}-x_2) - \{k_1\cdot(x_{d1}-x_2)+k_2\cdot(x_{d2}-x_1)\}$$

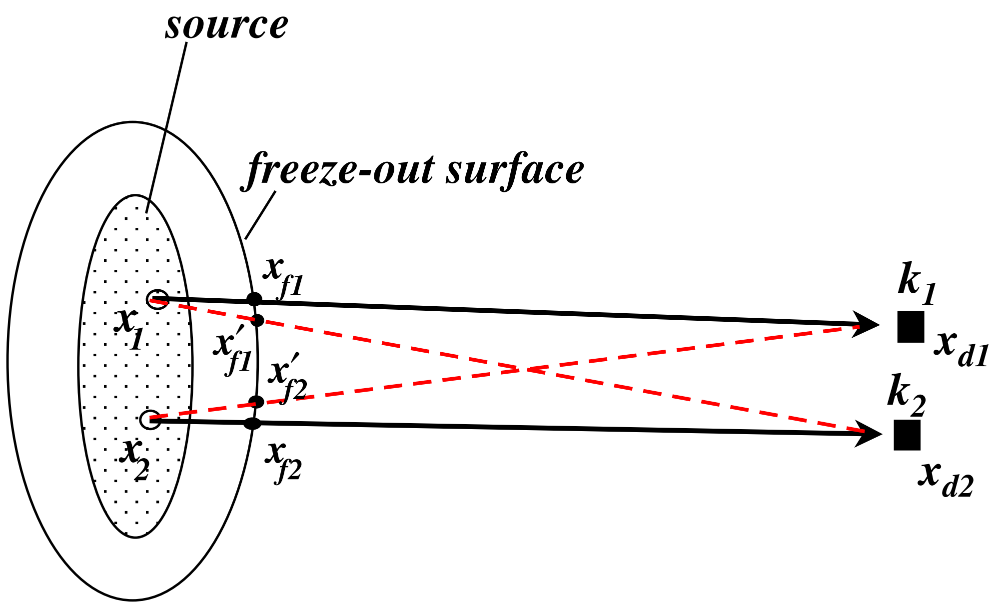
$$= -(k_1-k_2)\cdot x_1 - (k_2-k_1)\cdot x_2$$

$$R(k_1k_2) = \left(\sum_{x_1} A^2(k_1,x_1)e^{-i(k_1-k_2)\cdot x_1}\right)\left(\sum_{x_2} A^2(k_2,x_2)e^{i(k_1-k_2)\cdot x_2}\right)$$

$$\sim \left|\sum_{x_1} A^2(k_1,x_1)e^{-i(k_1-k_2)\cdot x_1}\right|^2$$

$$\sim \left|\int dx_1\rho(x_1)A^2(k_1,x_1)e^{-i(k_1-k_2)\cdot x_1}\right|^2$$

Conventional HBT Assumptions in Heavy-Ion Collisions



- As the detected particles traverse from the source point to the freeze-out point, they collide with medium particles.
- As a result of these random collisions with medium particles, the initial source will evolve into a chaotic source at freeze-out.
- The source that is observed in HBT measurements will be the chaotic freeze-out source, and the HBT radii will correspond to those of the freeze-out configuration.

Why we need to re-examine basic HBT assumptions

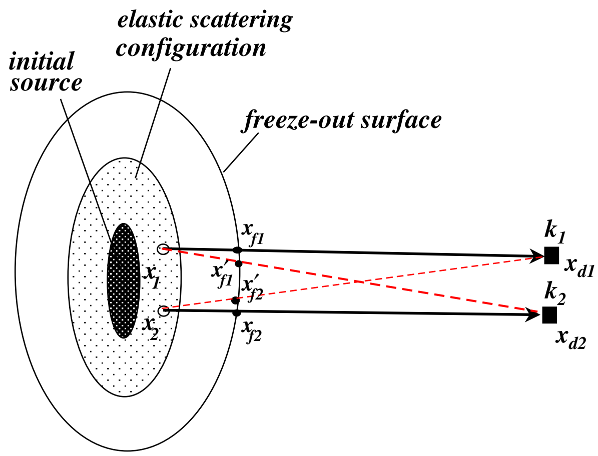
- Because the Hanbury-Brown-Twiss intensity interferometry is purely a quantum-mechanical phenomenon, the problem of multiple scattering must be investigated within a quantum-mechanical framework.
- It is necessary to study the interference of waves using the probability amplitudes in the multiple scattering process, instead of the conventional description of incoherent collisions in terms of probabilities and cross sections.
- The Glauber theory of multiple scattering and optical model have been shown to be a valid description for the interaction of a pion with the nuclear medium. They can be applied here to describe the probability amplitudes for the propagation of the detected particles.

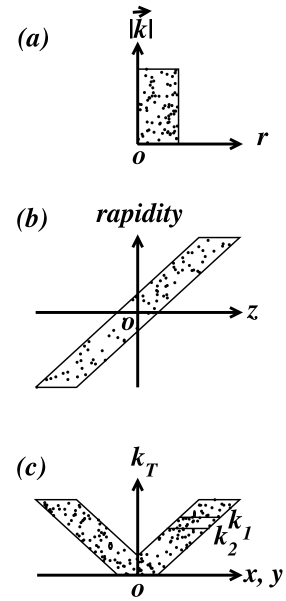
We find:

- HBT does not measure the freeze-out source distribution
- HBT measures an earlier pre-freeze-out density distribution — the *elastic-scattering* configuration

The *elastic-scattering* configuration of a source distribution is the configuration in which source particles begins to scatter predominantly elastically.

The *elastic-scattering* configuration occurs before, but close to, the state of chemical freeze-out.

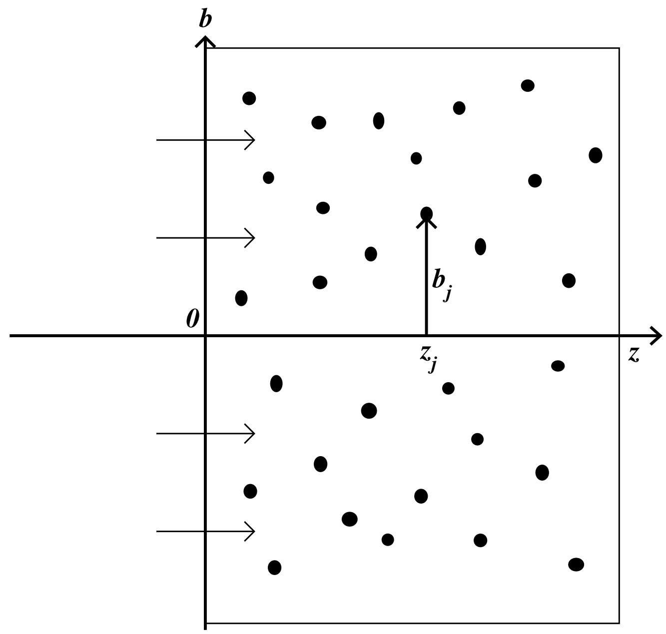




1. In a central RHIC collision, thousands of particle (mostly pions) are produced
2. Many initial collisions are inelastic 'chemical reactions' with a change of particle masses and flavors
3. The average kinetic energies (or the temperature) of particles subsequently decreases
4. Below a certain average kinetic energy, inelastic collisions stops and the system reaches the state of 'chemical freeze-out'
5. Subsequent collisions are elastic leading to a change of particle momentum distributions and the state of 'thermal freeze-out'
6. Particles are subject to collective longitudinal and transverse expansions

Glauber Theory of Multiple Scattering

Consider the propagation of a particle with momentum k along the z-direction through a slab of medium located at $z \geq 0$



According to Glauber theory, the phase of the wave function is the sum of the scattering phases of all its binary collisions. The wave function for propagation of the incident particle from 0 to the point $(\boldsymbol{b}z)$ is

$$\psi(0 \to \boldsymbol{b}z) \equiv \psi(\boldsymbol{b}z) = \exp\{ikz + i \sum_{j=1}^{N(z)} \chi_j(\boldsymbol{b} - \boldsymbol{b}_j)\}$$

$$= \exp\{ikz + i \ \ \phi(\boldsymbol{b}z)\}$$

What is the Phase Shift Function $\chi_j(\boldsymbol{b}-\boldsymbol{b}_j)$?

The phase shift function $\chi_j(\boldsymbol{b}-\boldsymbol{b}_j)$ can be obtained from two-body scattering data. It is related to the two-body profile function $\Gamma_j(\boldsymbol{b}-\boldsymbol{b}_j)$ by

$$\exp\{i\chi_j(\boldsymbol{b}-\boldsymbol{b}_j)\} = 1 - \Gamma_j(\boldsymbol{b}-\boldsymbol{b}_j).$$

The profile function can be expressed in terms of the two-body elastic scattering amplitude $f(\boldsymbol{q})$,

$$f(\boldsymbol{q}) = \frac{k}{2\pi i}\int e^{i\boldsymbol{q}\cdot\boldsymbol{b}}\Gamma(\boldsymbol{b})d\boldsymbol{b}.$$

Therefore, the imaginary part of the forward elastic scattering amplitude is

$$\mathcal{I}m\ f(\boldsymbol{0}) = \frac{k}{2\pi}\int \mathcal{R}e\ \Gamma(\boldsymbol{b})d\boldsymbol{b}.$$

From the optical theorem, we have the relation between $\mathcal{R}e\ \Gamma(\boldsymbol{b})$ and the two-body total cross section $\sigma_{\rm tot}$,

$$2\int \mathcal{R}e\ \Gamma(\boldsymbol{b})d\boldsymbol{b} = \sigma_{\rm tot}.$$

Single-Particle Wave Function $\psi(\boldsymbol{b}z)$

The wave function

$$\psi(\boldsymbol{b}z) = \exp\{ikz + i\ \phi(\boldsymbol{b}z)\}$$
$$\phi(\boldsymbol{b}z) = \sum_{j=1}^{N(z)} \chi_j(\boldsymbol{b} - \boldsymbol{b}_j)\}$$

from the multiple scattering theory contains a wealth of relevant information.

- It depends on the coordinates of all the particles with which the incident particles has interacted.
- It treats correctly the case of no scattering and multiple scattering, even up to the extreme case of $N(z)$ collisions in succession.
- It makes no difference whether the medium particles are dense in close proximity or dilute in far separation.
- Information on the density of medium particles can be provided when one integrates out the distribution of the medium particle coordinates.

The Wigner function

We introduce the Wigner function $w(\boldsymbol{q}z)$

$$w(\boldsymbol{q}z) = \int d\boldsymbol{s} e^{i\boldsymbol{s}\cdot\boldsymbol{q}} \; \langle\; \psi(\boldsymbol{b}+\frac{\boldsymbol{s}}{2}, z)\psi^*(\boldsymbol{b}-\frac{\boldsymbol{s}}{2}, z) \;\rangle_{\rm medium}$$

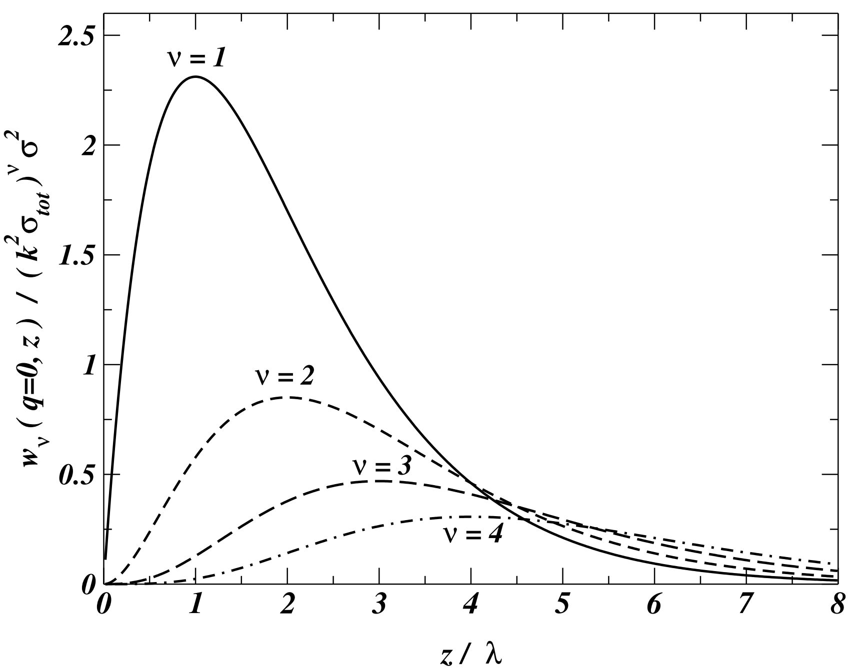
We can carry out the integration over $\boldsymbol{s}$. The Wigner function can be decomposed into multiple scattering components:

$$w(\boldsymbol{q}z) = \sum_{i=1}^{N(z)} w_\nu(\boldsymbol{q}z),$$

$$w_0(\boldsymbol{q}z) = e^{-nz\sigma_{\rm tot}}(2\pi)^2\delta(\boldsymbol{q}),$$

$$w_\nu(\boldsymbol{q}z) = e^{-nz\sigma_{\rm tot}}\frac{(2\pi)^2}{\nu!}\left(\frac{nz\sigma_{\rm tot}}{k^2}\right)^\nu \delta(\boldsymbol{q}-\sum_{j=1}^{\nu}\boldsymbol{q}_j)\int\prod_{j=1}^{\nu} d\boldsymbol{q}_j|f(\boldsymbol{q}_j)|^2.$$

$$\frac{d\sigma}{d\Omega} = |f(\boldsymbol{q}_j)|^2$$



Multiple Scattering Theory and Classical Transport

The transport equation for $w(\boldsymbol{q}z)$ can be obtained by differentiating $w(\boldsymbol{b}z)$ with respect to z. We find

$$\frac{\partial w(\boldsymbol{q}z)}{\partial z} = -n\sigma_{\rm tot} w(\boldsymbol{q}z) + n \int \frac{d\boldsymbol{q}_j}{k^2} |f(\boldsymbol{q}_j)|^2 w(\boldsymbol{q}-\boldsymbol{q}_j, z).$$

This is the same Boltzmann equation as one obtains in the classical case without using of the wave function.

- The multiple scattering theory leads naturally to the classical diffractive transport theory
- It retains the particle wave function
- It is therefore appropriate to use the multiple scattering wave function to investigate the effects of multiple scattering in interference phenomena such as the HBT.

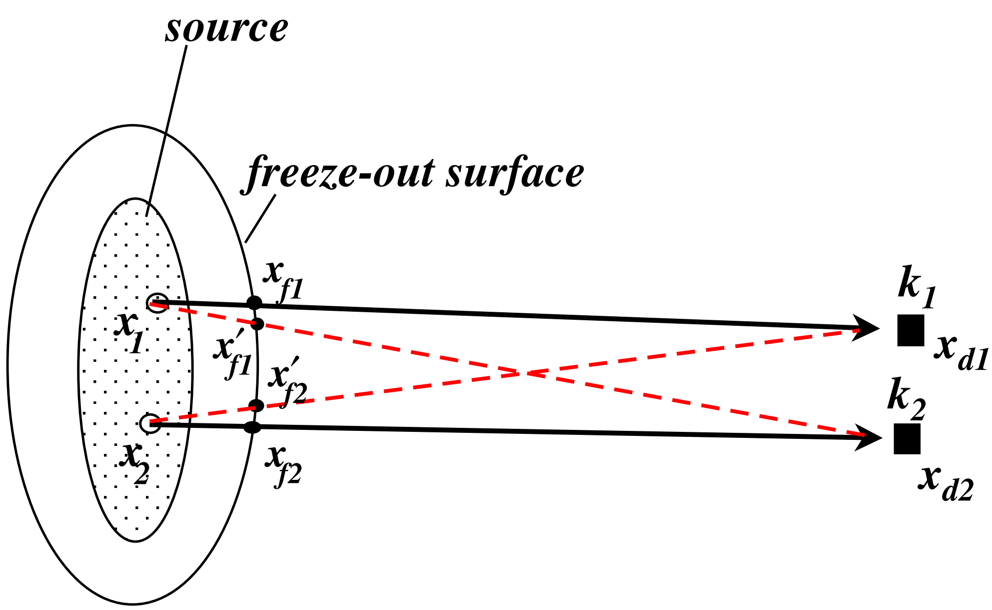
Multiple Scattering and Energy Loss

A particle loses its energy as it collides with medium particles. The energy loss can be treated in the multiple scattering theory by including the next-to-leading order term (Blankenbecler & Drell PRD53, 6265 (1996)):

$$\psi(\boldsymbol{b}z) = \exp\{ikz + i\phi(\boldsymbol{b}z)\}$$

$$\begin{aligned}
\phi(\boldsymbol{b}z) &= \phi^{(0)}(\boldsymbol{b}z) + \phi^{(1)}(\boldsymbol{b}z) \\
\phi^{(0)}(\boldsymbol{b}z) &= \sum_{j=1}^{N(z)} \chi_j(\boldsymbol{b} - \boldsymbol{b}_j) \\
\phi^{(1)}(\boldsymbol{b}z) &= -\frac{1}{2k} \int_0^z dz' |\nabla_\perp \phi^{(0)}(\boldsymbol{b}z')|^2
\end{aligned}$$

Application of Glauber Theory to HBT



Probability amplitude for particle with momentum k_1 to go from x_1 to the detector x_{d1}

$$\psi(x_1 \to x_{d1}) = \exp\{ik_1 \cdot (x_{d1} - x_1) + i\phi(x_{f1} - x_1)\}$$

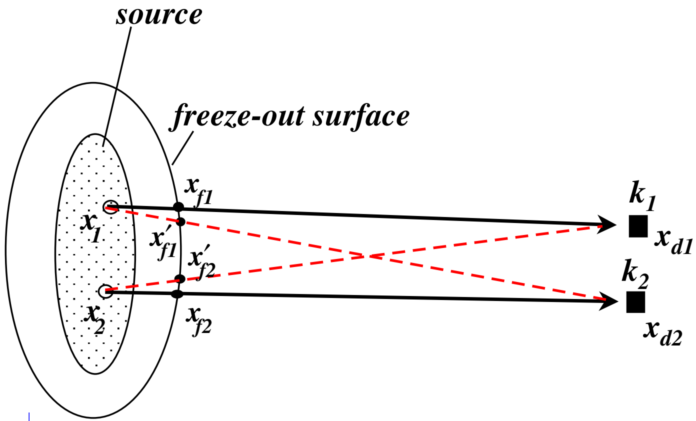
$\phi(x_{f1} - x_1)$ can come from Glauber multiple collision theory:

$$\phi(x_{f1} - x_1) = \phi^{(0)}(x_{f1} - x_1) + \phi^{(1)}(x_{f1} - x_1)$$

$$\phi^{(0)}(x_{f1} - x_1) = \sum_{j=1}^{N(z_{f1}-z)} \chi_j(\boldsymbol{b} - \boldsymbol{b}_j)\}$$

$$\phi^{(1)}(x_{f1} - x_1) = -\frac{1}{2k} \int_0^z dz' |\nabla_\perp \phi^{(0)}(x_{f1}{}' - x_1')|^2$$

HBT with Multiple Elastic Scattering



$$P(k_1,k_2) = \frac{1}{2!}\left|\sum_{x_1,x_2} A(k_1x_1)e^{i\phi_0(k_1x_1)}A(k_2x_2)e^{i\phi_0(k_2x_2)}\psi_{12}(x_1,x_2)\right|^2$$

$$= \sum_{x_1,x_2} A^2(k_1x_1)A^2(k_2x_2)|\psi_{12}(x_1,x_2)|^2$$

$$\psi_{12} = \frac{1}{\sqrt{2}}\Big\{ e^{ik_1\cdot(x_{d1}-x_1)+i\phi(x_{f1}-x_1)+ik_2\cdot(x_{d2}-x_2)+\phi(x_{f2}-x_2)} + e^{ik_1\cdot(x_{d1}-x_2)+i\phi(x'_{f2}-x_2)+ik_2\cdot(x_{d2}-x_1)+i\phi(x'_{f1}-x_1)} \Big\}$$

Phases cancel in the phase difference in HBT correlation:

$$k_1\cdot(x_{d1}-x_1)+\phi(x_{f1}-x_1)+k_2\cdot(x_{d2}-x_2)+\phi(x_{f2}-x_2)$$
$$-\Big\{k_1\cdot(x_{d1}-x_2)+\phi(x'_{f2}-x_2)+k_2\cdot(x_{d2}-x_1)+\phi(x'_{f1}-x_1)\Big\}$$
$$= -(k_1-k_2)\cdot x_1-(k_2-k_1)\cdot x_2$$

- For $R_{\rm out}$, $\boldsymbol{k}_1-\boldsymbol{k}_2$ is along $\boldsymbol{k}_1$, we have $\boldsymbol{x}_{f1}=\boldsymbol{x}'_{f1}$ and $\boldsymbol{x}_{f2}=\boldsymbol{x}'_{f2}$,

$$\phi(x_{f1}-x_1)=\phi(x'_{f1}-x_1), \quad \text{and} \quad \phi(x_{f2}-x_2)=\phi(x'_{f2}-x_2)$$

- For $R_{\rm side}$ and $R_{\rm long}$, $\boldsymbol{k}_1-\boldsymbol{k}_2$ is perpendicular to $\boldsymbol{k}_1$,

$$\phi(x_{f1}-x_1)-\phi(x'_{f1}-x_1) = (x_{f1}-x'_{f1})\cdot\nabla\phi(x'_{f1}-x_1)$$
$$\propto \boldsymbol{q}\cdot\nabla\phi(x'_{f1}-x_1)=0$$

$$R(k_1k_2) = \left|\int dx_1\rho(x_1)A^2(k_1,x_1)e^{-i(k_1-k_2)\cdot x_1}\right|^2$$

The HBT correlations will not be affected.

Effects of Multiple Scattering on HBT

$$\Psi(kx \to kx_d) = A(kx) e^{i\phi_0(x)} e^{ik\cdot(x_d - x) + i\ \phi(x_f - x)}$$

$$\phi(x_f - x) = \phi^{(0)}(x_f - x) + \phi^{(1)}(x_f - x)$$

$$\phi^{(0)}(x_f - x) = \sum_{j=1}^{N(z_f - z)} \chi_j(\boldsymbol{b} - \boldsymbol{b}_j)$$

$$\phi^{(1)}(x_f - x) = -\frac{1}{2k} \int_0^z dz' |\nabla_\perp \phi^{(0)}(x_f{}' - x')|^2$$

The imaginary part is given by

$$\mathcal{I}m\ \phi(x \to x_f) = \sum_j \mathcal{I}m\ \chi(\boldsymbol{b}_f - \boldsymbol{b}_j)$$

$$\begin{aligned} P(k) &= \sum_x |\ \Psi(kx \to kx_d)\ |^2 \\ &= \sum_x e^{-2\ \mathcal{I}m\ \phi(x)} A^2(k, x) \\ &= \int d^4x\ \ e^{-2\ \mathcal{I}m\ \phi(x)} \rho(x) A^2(k(x), x)\,. \\ &\equiv \int d^4x\, w(k, x), \end{aligned}$$

$$w(k, x) = e^{-2\ \mathcal{I}m\ \phi(x)} \rho(x) A^2(k, x).$$

$$P(k_1,k_2) = P(k_1)P(k_2)$$

$$+\,\theta\Big|\int d^4x e^{i(k_1-k_2)\cdot x} e^{-2\ \mathcal{I}m\ \phi(x)} \rho(x) A(k_1,x) A(k_2,x)\Big|^2.$$

The effective source density is given by

$$\rho_{\rm eff}(x;k_1,k_2) = \frac{e^{-2\ \mathcal{I}m\ \phi(x)}\rho(x)A(k_1,x)A(k_2,x)}{\sqrt{P(k_1)P(k_2)}}\,.$$

At lower energies, the interaction of the pion with the medium can be described by an optical model

$$\Psi(kx \to kx_d) = A(kx)e^{i\phi_0(x)}e^{ik\cdot(x_d-x)+i\ \phi(x_f-x)}$$

where $$\phi(x\to x_f) = -\int_{x_\parallel}^{x_{f\parallel}} \frac{1}{v} V(\boldsymbol{x}')dx'_\parallel$$

$$\mathcal{I}m\ \phi(x\to x_f) = -\int_{x_\parallel}^{x_{f\parallel}} \frac{1}{v}\, \mathcal{I}m\ V(\boldsymbol{x}')dx'_\parallel.$$

Conclusion

By applying the Glauber theory of multiple scattering to HBT, we find that multiple scattering leads to an effective density distribution that depends on a pre-freeze-out source distribution, the "elastic scattering configuration" of the source distribution.